Angular Momentum Regulation during Human Walking: Biomechanics and Control

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Abstract— Motivated by biomechanical studies on human walking, we present a control strategy for biologically realistic walking based on the principle of spin angular momentum regulation. Using a morphologically realistic human model and kinematic gait data, we compute the total spin angular momentum at a self-selected walking speed for one human test subject. We find that dimensionless spin angular momentum remains small ($S_j/$(Mass Height Velocity) < 0.02) throughout the gait cycle, and maximum whole body angular excursions within sagittal ($<1°$), coronal ($<0.2°$), and transverse ($<2°$) planes are negligible. These data support the hypothesis that spin angular momentum in human walking is highly regulated by the central nervous system, and that there exists a nonlinear coupling between ground reaction force, $\vec{F}$, center of mass position, $\vec{r}_{CM}$, and center of pressure location, $\vec{r}_{CP}$, or $\vec{F}=(F_z/z_{CM})(\vec{r}_{CM}-\vec{r}_{CP})$. We employ this relationship to rapidly generate biologically realistic CP and CM reference trajectories. Using an open loop optimization strategy, we show that biologically realistic leg joint kinematics emerge through the minimization of spin angular momentum and the total sum of joint torque squared, suggesting that both angular momentum and energetic factors are important considerations for biomimetic controllers.

Keywords- biomechanics; biped; locomotion; angular momentum; control; human.

I. INTRODUCTION

An important goal for biomimetic control systems is to generate reference motion trajectories that are consistent with underlying principles of biological movement. Generation of reference trajectories can be accomplished using optimization algorithms that take into account high level goals such as walking speed and direction, as well as biomechanical, energetic and physical objectives. It is highly desirable that this be accomplished quickly enough to be used in real-time control. The rapid generation of reference trajectories would allow the approach to be used for handling significant, unforeseen disturbances, enhancing the overall stability robustness of the system.

In this investigation, we seek underlying principles of human locomotory function that can be utilized by biomimetic control strategies to generate natural reference trajectories at low computational power. To this end, we hypothesis that spin

angular momentum in human walking is highly regulated by the central nervous system, and that there exists a nonlinear coupling between ground reaction force, $\vec{F}$, center of mass position, $\vec{r}_{CM}$, and center of pressure location, $\vec{r}_{CP}$, or $\vec{F}=(F_z/z_{CM})(\vec{r}_{CM}-\vec{r}_{CP})$. We test this hypothesis by comparing the forces predicted by the nonlinear model against ground reaction forces measured directly on a human walker using a force platform. Additionally, we compute the total spin angular momentum and the rotational effects of that momentum using kinematic gait data measured from a human test subject and a morphologically realistic human model. Finally, we further hypothesize that both angular momentum and energetic factors are important considerations for biomimetic controllers. Using an open loop optimization strategy, we test whether biologically realistic leg joint kinematics will emerge through the minimization of spin angular momentum and the total sum of joint torque squared.

II. ANGULAR MOMENTUM REGULATION: BIOMECHANICS

A. Spin Angular Momentum and Resulting Whole Body Angular Excursions during Human Walking

Biomechanical investigations [1,2,3,4] have determined that a large class of human movements, including standing, walking and running, support conservation of total angular momentum, $\vec{S}$, about the body's center-of-mass (CM), or

$$\frac{d\vec{S}}{dt}\Big|_{CM} = 0. \tag{1}$$

Angular momentum is a conserved physical quantity for isolated systems where no external moments act on a body's CM. However, in the case of legged locomotion, where the body interacts with the environment (ground reaction forces), there is no *a priori* reason for this relationship to hold. It is asserted here that spin angular momentum is highly regulated ($\vec{S}\approx0$) by the central nervous system *throughout a movement cycle*.

For this investigation, spin angular momentum was calculated for human walking at self-selected speeds using a morphologically realistic model shown in Fig.1. The model, consisting of 16 links, has 32 degrees of freedom. Using human morphological data from the literature and direct

measures on the human test subject, each link's dimensions and mass densities were carefully modeled to achieve realistic mass distributions.
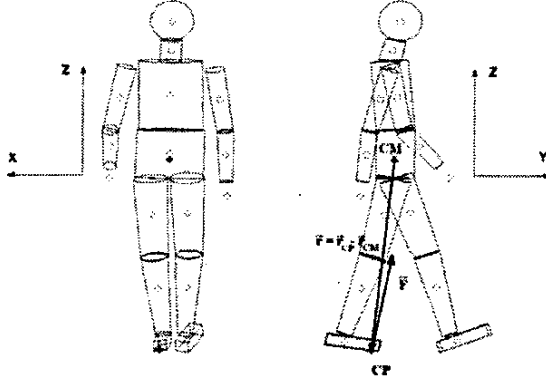

Figure 1: Morphologically realistic human model.

In Fig. 2A, a dimensionless spin angular momentum, defined as the angular momentum, S_i, divided by body mass, M, height, H, and walking speed, V, is plotted versus percent gait cycle for one human test subject (M=50.1Kg; H=158cm; V=1.3m/sec). By convention, 0% and 100% represent consecutive heel strikes of the same foot. The average (solid line) and standard deviation (dashed lines) are plotted for seven walking trials. The resulting dimensionless spin is surprisingly small; throughout the gait cycle, none of the three spatial components ever exceed 0.02 dimensionless units.

To determine the effect of the non-zero angular momentum components shown in Fig. 2A on whole body angular excursions, we first computed the whole body angular velocity vector and then the corresponding angular excursions. These quantities are defined as

$$\vec{\omega}_{eff} = \bar{I}^{-1}\vec{S} \quad \text{and} \quad \vec{\theta}_{eff}(t) = \int_{-\infty}^{t} \vec{\omega}_{eff}(t^*)dt^* + C \quad (2)$$

respectively, where $\bar{I}$ is the whole body moment of inertia tensor about the CM and C is an integration constant determined through an analysis of boundary conditions. The whole body angular excursion vector defined in equation (2) can be accurately viewed as the rotational analog of the CM position vector. The results of these analyses, shown in Fig. 2B, show that the maximum whole body angular excursions within sagittal (<1°), coronal (<0.2°), and transverse (<2°) planes are negligibly small. As in Fig. 2A, 0% and 100% represent consecutive heel strikes of the same foot, and the average (solid line) and standard deviation (dashed lines) are plotted for seven walking trials. These results support the hypothesis that spin angular momentum in human walking is highly regulated by the central nervous system so as to keep whole body angular excursions at a minimum.
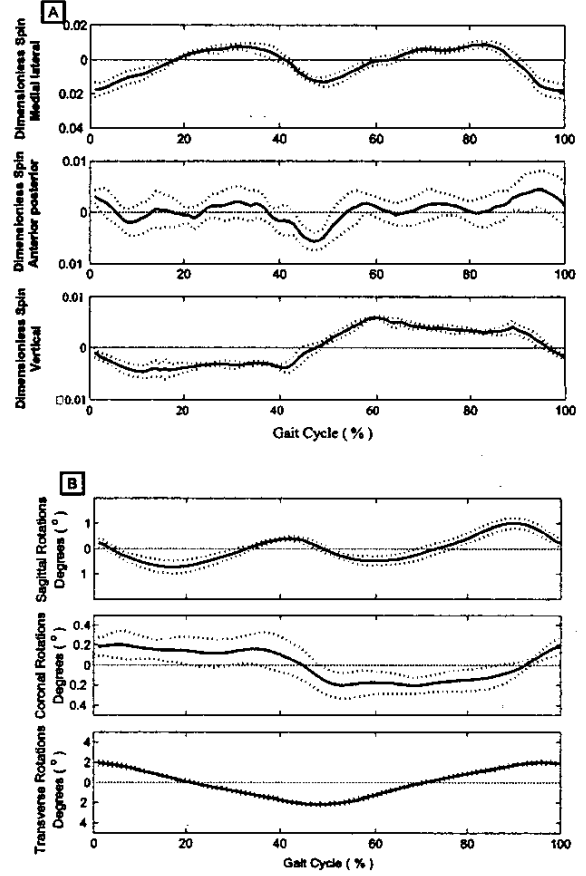

Figure 2: Normalized spin angular momentum (A) and whole body angular excursions (B) for body rotations within sagittal, coronal and transverse planes.

B. Spin Angular Momentum Regulation: The CM Force, CM Position and CP Position Non-Linear Coupling

The total torque about the body's CM may be expressed as

$$\vec{T}_{CM} = (\vec{r}_{CP} - \vec{r}_{CM}) \times \vec{F} = \frac{d\vec{S}}{dt} \quad (3)$$

where $\vec{r}_{CP}$ is the center of pressure on the ground, or zero moment point (ZMP) in the robotics literature [5], $\vec{r}_{CM}$ is the CM position, and $\vec{F}$ is the ground reaction force (GRF) vector. Solving for the transverse component of the GRF vector gives

$$F_Y = \left[(\delta y)F_Z - T_X\right]/(\delta z) \quad \text{and} \quad F_X = \left[(\delta x) F_Z + T_Y\right]/(\delta z) \quad (4)$$

where $\delta\vec{r} = \vec{r}_{CP} - \vec{r}_{CM}$.

Angular momentum regulation throughout a movement cycle requires that the sum of torques about the CM is always equal to zero. This condition gives a nonlinear relationship between the body's CP, CM, and the ground reaction force vector $\vec{F}$, or

$$\vec{F} = k\,\delta\vec{r} \qquad (5)$$

where $k = F_Z / \delta z = -F_Z / z_{CM}$ is a global body stiffness.

Equations (4) offer an effective way to assess the degree to which spin angular momentum is regulated during any human movement task; the first term within the brackets on the RHS of each equation (4) represents a natural human dynamics reference value against which the second, non-conservation term may be compared.

Using equation 5, the CM and CP positions, $\vec{r}_{CM}$ and $\vec{r}_{CP}$, together with the global stiffness, k, are used to predict the transverse forces that act on the body's CM during human walking. The ground reaction forces and CP locations were measured directly using a force plate (model OR6-5-1, Advanced Mechanical Technology, Newton, MA), and the CM location was computed using the morphologically realistic human model (Fig. 1) where the limbs of the model were moved through a walking cycle as defined by motion capture data (VICON 512, 120 frames/sec). The transverse forces predicted using equation 5 were then compared to the experimentally measured ground reaction forces to further test the validity of the spin angular momentum regulation hypothesis.

In Fig. 3, the model (thick solid line) is in good agreement with the force plate data (thin solid line). Here 0% to 50% gait cycle spans from the middle of a single support phase to the middle of the next single support phase of the opposite limb, and the averages (solid lines) and standard deviations (dashed lines) are plotted for seven walking trials. The R^2 value across 7 experimental walking trials is 0.98±0.02 for the medial-lateral forces and 0.94±0.03 for the anterior-posterior forces (a value of 1.0 corresponds to perfect agreement). These high R^2 values lend additional support to the hypothesis that spin angular momentum is highly regulated during human walking.
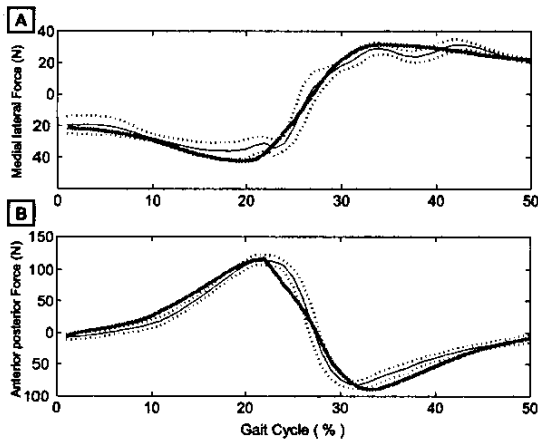

Figure 3: Spin model force predictions for medial-lateral (A) and anterior-posterior (B) directions during walking.

A similar expression to equation (5) was obtained by [6] when analyzing the mechanical properties of a 3D Linear Inverted Pendulum Mode (LIPM) based model. The LIPM model constrains the CM to move in the transverse plane and

the base of the inverted pendulum is held *fixed* to the ground. In principle, this model is a special case of the regulated spin model presented in this paper. By constraining the CM to move in the transverse plane, the global body stiffness, k, defined in equation (5), is held constant throughout the gait cycle and is set equal to body weight divided by the constant vertical height of the CM. Neglecting the CP acceleration term in the LIPM force prediction, as was done by [6,7,8,9,10,11], we compare the transverse force predictions of the more general nonlinear spin model to the linear (constant k) LIPM model. The regulated spin model generally gives better force predictions throughout most of the gait cycle (85%). Surprisingly, the LIPM model gives a better prediction in the double support phase as compared to the single support phase; in the middle of the single support phase (labeled by 0% and 50% in Fig. 3) its medial-lateral force prediction is off by more than 60% (the linear LIPM model predicts a force equal to about 33 N while both the measured and the nonlinear spin model predicts a force equal to about 20 N). We conclude that the nonlinear spin regulation model (equation 5) is a more accurate representation of human walking than the special case, linear LIPM model.

C. Spin Angular Momentum: Non-Regulation

Given the CM position and the ground reaction force, the position of the CP can easily be computed using equation (5). We call that point the *zero spin* CP point. Clearly, the actual CP of biological or robotic systems will differ from the *zero spin* CP if spin angular momentum is not precisely regulated. Significant separation distances between the *zero spin* CP and the actual CP are expected for a class of human movement tasks where:

1) the ground reaction force is so large that the *zero spin* CP point moves outside the foot support polygon; or

2) large and rapid turning motions occur (such as would result from a non-zero vertical torque) moving the *zero spin* CP point away from the actual CP location.

In the first case, the switching of control strategy or mode from regulation to non-regulation was recently observed [4]. For hip swiveling motions about the vertical axis while in standing double support, it was observed that while the *zero spin* CP was confined within the foot support polygon, the actual CP was found to track the *zero spin* CP with very high precision. However, when the hip rotational velocity became sufficiently large, the *zero spin* CP left the foot support polygon, and the actual CP moved from the outside edge of the foot-support polygon towards its center.

In the second case, for example from a non-regulated vertical spin component, or

$$T_{CM\,z} = (\delta x)\ F_y - (\delta y)\ F_x = \frac{dS_z}{dt} \neq 0 \qquad (6)$$

it can be inferred that the transverse spin component will also not be regulated.

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III. Angular Momentum Regulation: Control

A. Rapid CP and CM Trajectory Generation using the Principle of Spin Angular Momentum Regulation

An important goal in humanoid robotic locomotion is generation of reference motion trajectories based on high-level goals like direction and speed of walking. Robust controllers can then be used to track these reference trajectories to achieve stable walking in the presence of disturbances.

Generation of reference trajectories can be accomplished using optimization algorithms that take into account the high level goals, as well as biomechanical, energetic and physical objectives. It is highly desirable that this be accomplished quickly enough to be used in real-time control. The rapid generation of novel reference trajectories would allow the approach to be used for handling significant, unforeseen disturbances, enhancing the overall stability robustness of the system.

The reference trajectories must be dynamically feasible; it must be possible for the combined control system and plant to achieve the reference trajectories. The combined system is subject to numerous physical constraints such as actuation force and bandwidth limitations, sensor noise and joint range limits. In the case of biomimetic control systems, the reference trajectories should also be consistent with underlying principles of human locomotory function. This is especially important for assistive devices like exoskeletons, powered orthotics, and powered prosthetics. The motion of such devices should feel as natural to the user as possible. Biomimetic controllers grounded in principles of biological movement are also important for animats and humanoid robots; it is desirable that such robots move in an animal or human-like manner.

Generation of biologically realistic trajectories was recently achieved using a dynamic optimization approach [12]. A 23 degree-of-freedom model actuated by 54 muscles was used in this study. This model, with its associated kinematic and dynamic constraints, was input to a dynamic optimization algorithm, along with the high-level goal of minimizing metabolic energy expenditure per unit distance traveled. The dynamic optimization yielded biologically realistic joint motion and force trajectories.

This result, although elegant scientifically, is not practical for real-time control; computation required close to one hour on a Cray supercomputer. Fortunately, it is possible to use much simpler models for generation of reference trajectories [8, 9, 11]. These approaches appear to be promising for real-time control, but they have not been evaluated in terms of biological realism. Thus, the open question with the simple model approach is whether it is fast enough for use in real-time control while also producing trajectories that are consistent with biomechanical and energetic principles of human movement.

We propose a methodology for generating reference trajectories consistent with the model of spin angular

momentum regulation, or equation (5). Equation (5) can be used to predict CP from a known CM trajectory, as is shown in Fig. 4A. The heavy dashed line is the predicted CP, the solid gray line is the corresponding measured value for CP from biological trials, and the dotted lines indicate standard deviation of the biological measurements. Note that the predicted CP trajectory agrees well with the biological CP trajectory measured from a force platform. Fig. 4A represents an average over 6 trials with one test subject. Note that the CP prediction also gives a good indication of foot placement. This is useful for deciding foot placement after a disturbance to CM when in single support.

The principle of spin angular momentum regulation can also be used to predict horizontal components of the CM from a known CP trajectory. This approach requires input of the global body stiffness, k, defined in equation (5). Equation (5) then becomes a non-liner differential equation in CM which can be solved using numerical integration algorithms. Values for k for normal walking can be obtained from biological data or from a reduced order walking model. In certain cases, it is possible to set k equal to a constant based on the morphology of the test subject. With this assumption, equation (5) becomes a linear differential equation in CM which can easily be solved. This works well for double support, but can lead to significant errors during single support due to the reasons previously discussed in section IIB.

Given the ability to predict horizontal components of CM from CP, the following procedure can be used to generate reference trajectories. We begin with a specification of walking speed and direction. From this, we generate desired gait length and width, taking into account the morphology of the system. We then generate foot placement, and from this, a CP trajectory that stays within the support polygon. The corresponding horizontal components of the CM trajectory are then generated by integrating equation (5). Once both CM and CP trajectories are known, trajectories for joints are generated, as will be discussed in the next section.

Besides a prediction of CP from CM and vice-versa, we can also use spin regulation to predict how the CP and CM trajectories might change after a disturbance. Disturbances can be classified according to their effect on CM or CP. For example, a push (an un-anticipated external force acting on some part of the body) can be modeled as an impulse force on the CM, resulting in an instantaneous change in velocity of the CM. A slip (un-anticipated translation of a stance foot in the ground plane) can be modeled as an instantaneous change in velocity of the CP. Similarly, a roll (un-anticipated rotation of foot) can be modeled in this way.

When a disturbance occurs, it is very useful to predict the resulting time evolution of CM and CP using equation (5). This is useful for planning corrective actions that may be necessary. To understand this, it is important to be clear about the distinction between a number of commonly used terms in the literature. The CP (a.k.a ZMP, as defined in [5]) is the center of pressure within the support polygon. This can be measured using force plate data in biological tests, and it can

be computed in various ways [13]. The FRI (Foot Rotation Indicator, [13]) is defined as the point on the foot/ground contact surface where the net ground reaction force would have to act to keep the foot stationary. If the FRI extends beyond the actual support polygon, the foot begins to roll. *Zero spin* CP, a term we introduce here, is computed by equation (5). Like FRI, it is not limited to the actual support polygon. Because it is computed by equation (5), *zero spin* CP does not, necessarily, correspond to CP or to FRI. However, as explained in preceding sections of this paper, the human central nervous system seems to control normal walking such that the *zero spin* CP largely coincides with CP, and both remain within the bounds of the support polygon.

This situation changes when there are sufficiently large disturbances. For such disturbances, the human control system must fundamentally change its control mode, because, remaining in the zero spin control mode would result in the *zero spin* CP leaving the support polygon. The new control mode required to compensate for such a case requires non-zero spin, which is accomplished by appropriate extensions of the arms, swing leg, and trunk. Thus, when a disturbance occurs, a control system must decide whether to switch to the new control mode. One way to do this, immediately when the disturbance occurs, is to predict the future time evolution of CM and *zero spin* CP, assuming the system remains in zero spin control mode. If the resulting *zero spin* CP leaves the support polygon, the system knows that it will have to switch modes long before this exit of the *zero spin* CP occurs. Thus, the system has enough time to switch control modes and compensate before stability problems occur.

The simultaneous prediction of CP and CM can be accomplished beginning with nothing more than a few initial conditions on position and velocity of CP and CM, and a few very simple final conditions on velocity (but not position). Two additional simplifications are used for this prediction. First, a constant value of k is used. This is justified, in this particular case, because the force errors due to this assumption are limited to a small part of the gait cycle, and so, the double integration in equation (5) results in a relatively small accumulated error of CP. Also, the goal of this prediction is to determine, roughly, whether CP will exit the support polygon, or come close to its edge; exact position of CP is less important. The second simplification, based on observations of human test subjects during normal walking, approximates the CP trajectory as a straight line going from the back of the foot to the front during single support, and as a straight line going from the toe of the trailing foot to the heel of the leading foot during double support. This allows the CP trajectory to be represented using only a few parameters. With this simplification, and with the above-mentioned boundary conditions, equation (5) can be solved as a boundary value problem to generate both CM and CP trajectories. This is shown in Fig. 4B for normal walking, from the middle of double-support, to the middle of single-support. As is shown, the model prediction (heavy dashed line) agrees well with the biological data (solid gray line). As before, the dotted lines represent the standard deviation of biological data. Note that the CP plot ends approximately at the middle of the foot,

indicating its position at the middle of single-support. Fig. 4C shows a similar plot, but with an impulse disturbance to the CM at the middle of the double-support phase. This changes the CM initial velocity boundary condition (in this case, by a factor of about 1.3), resulting in a deflection of the CM and CP trajectories.
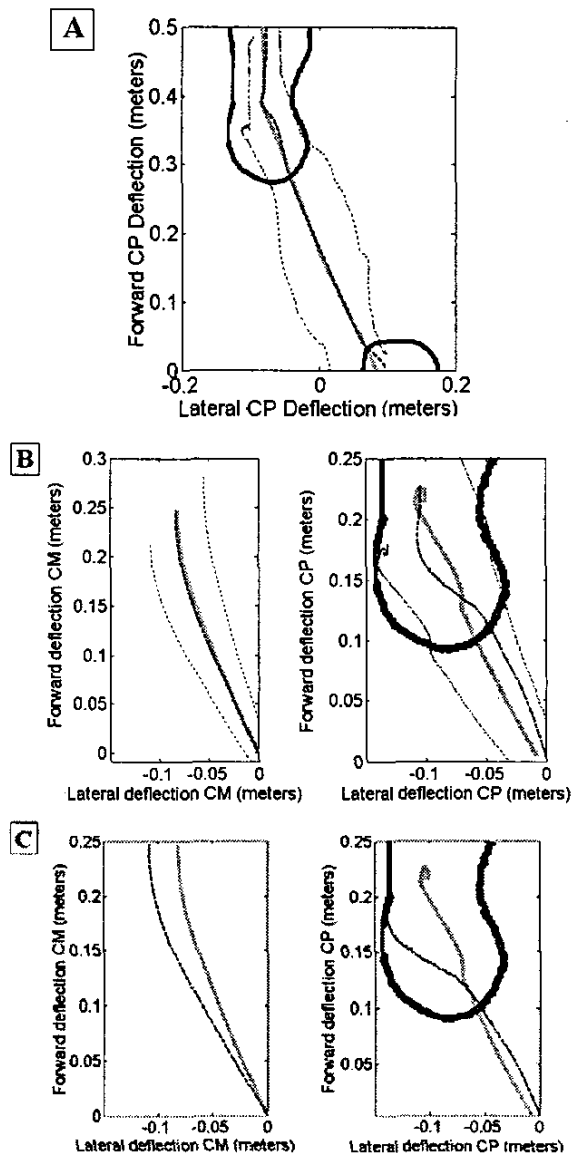

Figure 4: Rapid CP and CM trajectory generation based on spin angular momentum regulation.

The disturbance results in the CP being deflected to the edge of the foot at the middle of single-support. If the disturbance were greater, the *zero spin* CP would extend beyond the support polygon, and equation (5) would no longer be valid as a control law. In this case, non-zero spin must be asserted by the control system to restore stability. This

accounts for some human behaviors such as bending the trunk, and moving the arms and swing leg out in order to restore balance. The ability to predict the exit of the *zero spin* CP from the support polygon before it actually occurs is important for planning corrective action.

B. Open Loop Optimization: Angular Momentum and Joint Torque Squared Minimizations

Once both CM and CP trajectories are known, it is possible to generate trajectories for joints using, primarily, kinematic constraints. We use a space-time optimization technique [14] with a reduced-order five-link sagittal plane model. The links for this model are stance foot, stance lower leg, stance thigh, body, swing thigh, and swing lower leg. Joints are stance ankle, stance knee, stance hip, swing hip, and swing knee. Although it is a simplified model, link inertias and dimensions are set to correspond to those of the human test subject. The body link combines the inertias of the human test subject's trunk, head, and arms.

The reduced-order model is used to compute forward kinematics (Cartesian positions of links based on joint angles), and also inverse dynamics according to

$$\tau = h(\ddot{\theta}) + c(\dot{\theta}) + g(\theta) \qquad (9)$$

where τ is the joint torque vector, θ is the joint angle vector, and h, c, and g are functions that compute inertial, velocity cross-product, and gravitational terms.

Joint trajectories are represented using splines with 10 control points, and are constrained so as to conform, within tight limits, to the reference CM and CP trajectories. The optimization cost function penalizes non-regulated spin angular momentum, and large joint torques. Resulting trajectories are shown in Fig. 5.

This approach depends, largely, on the accuracy of the input CM and CP trajectories (which are computed as discussed in the previous section). These significantly constrain the joints, and this allows for fast generation of accurate joint trajectories. Note that with this approach, correct behavior of the swing leg emerges automatically; it need not be explicitly specified. This is in contrast to the approach used in [15], which requires explicit specification of swing foot trajectory.

IV. CONCLUSION

Using a morphologically realistic model and kinematic gait data for normal walking from a human test subject, we find that spin angular momentum remains small throughout the gait cycle. This supports the hypothesis that spin angular momentum in human walking is highly regulated and leads to a non-linear coupling between ground reaction force, CM and CP positions. We show that this nonlinear spin regulation model can be used to quickly generate biologically realistic reference trajectories for normal walking. We also use this relationship to determine whether a disturbance is sufficiently large to necessitate switching to a new control mode; a controller where spin angular momentum is no longer regulated. The reference trajectories for CM and CP can be used as inputs to a space-

time optimization algorithm, which, when biased by a cost function that minimizes spin angular momentum and joint torques squared, generates biologically realistic joint trajectories. In particular, the swing leg trajectory emerges automatically, and need not be specified explicitly. In the advancement of biomimetic control systems, we feel that the minimization of both spin angular momentum and energetic factors are important design considerations.
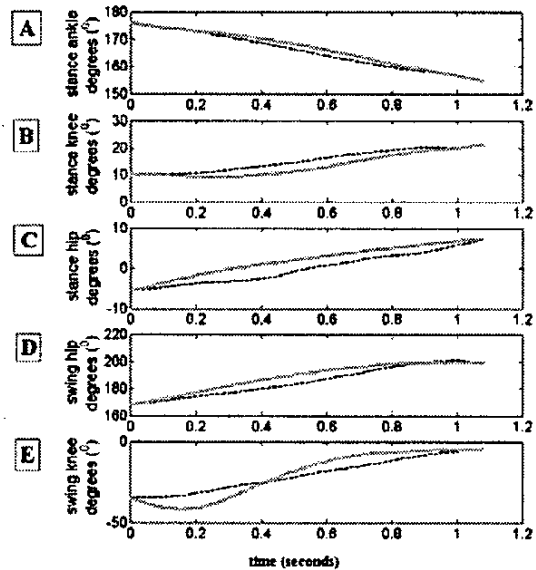

Figure 5: Model predictions of ankle, knee and hip joint angle trajectories during walking using spin regulation and joint torque minimization. Solid gray line shows model prediction, dotted black line shows biological trajectory.

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